Ornamental plate shell structures

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Abstract: The development of digital technologies in the last twenty years has led to an unprecedented formal freedom in design and in the representation in virtual space. Combining non-standard geometry with CAD tools enables a new way of expression and realization of architectural ideas and conceptions. But the transformation of a virtual double-curved surface into a buildable physical structure and object is always accompanied by huge costs and big problems like geometric and statical ones. With this work we propose a method how to transfer a double curved surface into a cost-efficient buildable shell structure consisting of planar building elements derived from tangent planes and based on ornamental discretization. This approach should also serve as a geometric basis for an interface whereby a user can transfer his designed ornamental 2D-pattern onto a desired freeform. The novel process in our work is that we take the surface curvature at local points into account. This solves former problems which occurred when intersecting the tangent planes. Additionally local control of the spatial ornamental structure is provided.

The load bearing system is organized in a way so that the forces are distributed along the edges of the plane elements. A structure with plane elements supports a high stiffness in combination with a relatively small overall weight. This is due to the smooth curved shape of the geometry.

1. Introduction

Unconventional geometric shapes and free-form surfaces - also known as non-standard geometry - have always been something that architects wish to design and build [Farin 2002, Kolarevic 2003, 2008]. The first step in the creation of buildable free forms in architecture is their discretization. The discretization of free-form surfaces is defined as the process of partitioning a continuous surface into discrete counterparts. This process is usually carried out as the first step toward determining those parts of a surface that make a surface suitable for numerical evaluation and calculation.

Free-form surfaces may be discretized in a number of ways and in accordance with a number of principles. One method of surface discretization results in planar panels, whereas with another curved segments are obtained. In terms of geometry, there are two fundamental differences between these two methods of discretization. First, when a surface is discretized into planar elements, the initial surface form is approximated in order to obtain planar elements, and depending on the size of individual elements, a greater or lesser distortion in the geometry of the set form occurs. With the second method of discretization, the set form is not approximated; instead, the complete form is partitioned into smaller curved segments. When a surface is discretized into planar elements, the obtained segments of the surface share straight boundary edges, while with curved segments the boundary edges are curved.

Nevertheless, if a surface has to be discretized in view of its actual construction, obviously from the aspect of technology discretization into planar elements is both more feasible and cost-effective, in comparison with discretization into curved elements.

In our work we would like to give a designer the opportunity to choose the form and size of discretization he needs. Our approach is the way of using plane ornaments as a 2D starting point which can be manipulated by the user and afterwards mapped automatically onto a 3D surface. This mapping will be described in this paper. The designing of the user interface is another part of our project which is work in progress. Figure 1 shows an example of an ornamental cell which can be controlled by the user by moving and mirroring it in the 2D-plane (two pictures on the left side). On the right side a freeform surface and the mapping of the ornament onto it can be seen.

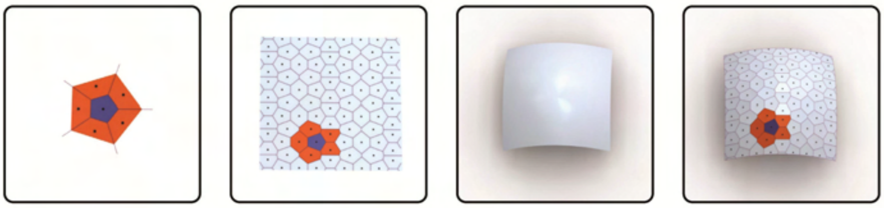

Fig.1 Ornamental cell, 2D ornament generated with the cell (moving and mirroring), freeform surface and the mapping of the ornament onto the surface

2. Discretization

2.1 Discretization into curved segments

In this particular case, it is possible to partition the selected free-form surface according to an arbitrary panelization scheme, hereinafter referred to as the ornament, which would serve as a pattern for the discretization of the complete surface. It may be said there is an infinitely great number of ornaments in the wall ornament group [Shubnikov 1974, Quaisser 1994, Moussavi 2006], which can be used to discretize the selected surface. The upside of this method is that the surface is segmented within the *u* and *v* parameters and the selected 2D ornament can easily be projected onto a curved free-form surface. In that case, depending on the type of surface, the selected 2D pattern will undergo some deformation, elongation or contraction.

Attempts to materialize this kind of geometric model using real building materials reveal the downsides of this method. In building construction, it is possible to construct double curved surfaces using a combination of steel and glass (a steel frame in combination with glazed panels), or concrete. Currently, the manufacturing of double curved glass has reached a point where it is necessary to produce molds first which correspond to the required curved form, which leads to extremely high costs and makes the construction of such buildings cost-ineffective. When concrete is the preferred material, the problems of formwork construction and reinforcement placement make building complicated and costly [Sauter 2008]. It may be said the economic aspect is not the crucial one when it comes to big construction projects, but the question remains as to how buildings with double curved surfaces can be constructed in a feasible and economical way. This has been a debating point in glass and concrete industry, and future research might lead to the creation of new, cost-effective technologies that will be able to allow the materialization of such complex forms.

One possible way to construct such buildings using currently available technologies is by re-geometrizing geometric forms. The term re-geometrization means the selection of such geometric forms that may be discretized into “similar” curved segments. It would be possible to construct such forms using a small number of such elements, which would considerably reduce the production cost of double curved surfaces, glass or concrete.

Another issue of concern when it comes to this type of discretization is the joining technique. In the case of discretized elements with curved boundary lines, the nodes are under torsional moments, making the calculations more difficult to carry out.

2.2 Discretization into plane segments

A surface can be partitioned into planes using different techniques, triangulation, quad meshing, or freely placed tangential planes. There are both advantages and disadvantages to each of these methods of discretization, which will be discussed briefly.

Triangulation is the best-known method of curved surface discretization (Fig. 2). This method is used for partitioning a selected surface into triangular planar segments. The drawback of this method is a very large number of edges characterized by a high degree of geometric complexity, which in turn requires many load-carrying members, large quantities of structural materials, and increases construction costs. When it comes to the aesthetics, only the size and aspect ratio of triangular panels can be influenced.

Fig. 2 Triangulation – Murinsel Graz

The second method is quad meshing, where a surface is divided into quadrangular polygons by means of tessellation [Sauer 1970, Glymph et al. 2002, Liu et al. 2006, Pottmann et. al. 2008, Zadravec et al. 2010]. From the aspect of use of materials, surface tessellation is more optimal than tessellation into triangular elements. However, it cannot be easily employed with arbitrary surfaces.

The third type of tessellation of free-form surfaces is planar tessellation, where plane polygons are obtained. The construction is based on the duality between the surface points and their tangent planes. The tessellation works on the intersection of numerous tangent planes of the selected surface. If the tangent planes of a smooth surface are positioned in a certain way, it is possible to obtain convex hexagonal “honeycomb” panels. This is achievable if the observed surface is positively curved. If the surface is negatively curved, we get butterfly-formed panels. This topic is discussed for instance in Wang (2008) and Wang (2009). If tangent planes are selected at consecutive points on the maximal and minimal curvature lines, it is possible to predict the orientation of the intersecting edge between two adjacent surfaces. Following this principle, the paper [Troche, 2008] also uses a TPIAFT algorithm (Tangent Plane Intersection – Advanced Front Technique) to panelize a positively curved surface and optimize the size and shape of the panels, although without applying the aesthetic principle.

The fact that there is an infinite number of possibilities when selecting points and their tangent planes on a surface (Fig. 3, Fig. 4) raises the issue of the way and conditions which make it possible to select specific tangent planes whose intersection would produce the desired shape in accordance with the previously selected tessellation, a 3D ornament. Another issue is whether there is an infinite range of possibilities to generate a preferred 3D ornament and on what conditions surface tessellation would be ornamental in character, i.e. it would produce not only the functional, but also the aesthetic component of a free-form surface.

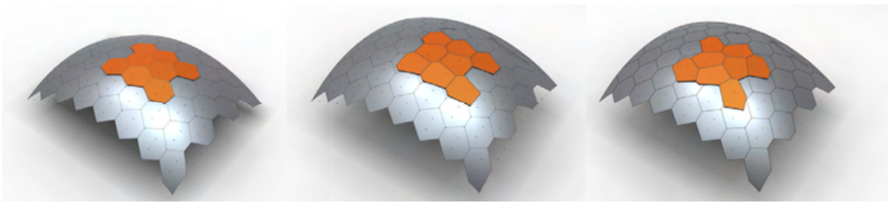

Fig. 3 Different choices of tangent planes produce different plane panels

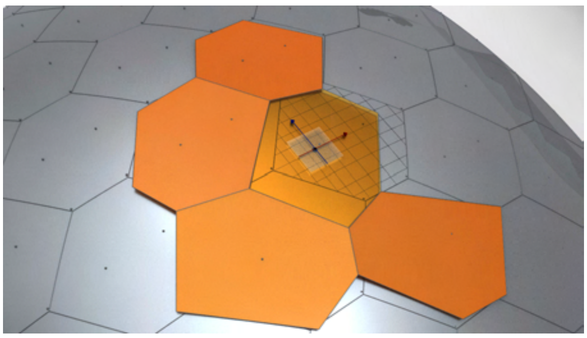

Fig. 4 Panels can be modified by changing the *uv* parameters of the associated point on the curved surface

This paper presents a new simple and geometric approach to panelizing free-form surfaces, with the goal of generating panels according to specific aesthetic criteria. This analysis is based on previously solving the problem of TPI (tangent plane intersection) on an arbitrary curved surface. The construction takes also the curvature properties of the given free form surface into account and provides local control. Local control is a great advantage over constructions where optimization algorithms are used “to design our design”. So we can interfere in regions of our planarization where a special design is needed and desired.

The main problem in the process of discretization of free-from surfaces is choosing the correct adjacent tangent planes that model the polygonal panels and finding the orientations of their edges.

3. Planarization

Our approach to the discretization of a smooth double curved surface S is based on the duality between surface points and their tangent planes (see e.g. Troche (2008), Wang (2008), Wang (2009)). We take a number of surface points P_i $(i = 1, ..., n)$, which (at this stage) are arbitrarily distributed on S. We make sure the points P_i are either elliptic or hyperbolic and avoid parabolic points [Do Carmo 1976, Farin 2002]. To every point P on S we determine its tangent plane τ_P and intersect it with the tangent planes of the adjacent points (Fig. 4). We keep that part of τ_P which encloses P. This part is convex if P is an elliptic surface point and concave if P is hyperbolic or parabolic. An important question is the choice of points adjacent to P. One known approach is to use a Delaunay triangulation to Fig. out the nearest and neighboring points. This can be done by a triangulation of the parameter value set (u_i, v_i) of the set P_i in the $[u,v]$-plane or as a spatial 3D Delaunay triangulation of the points P_i. This works in some cases but often causes problems (see Troche 2008 and Fig. 4). This is due to the fact that neither is an adapted surface measurement in the form of geodesic lines used, nor is the curvature of the involved surface taken into account. This kind of use of triangulation will only yield accurate results for those parts of a surface where there is not a great curvature variance, i.e. where the principle curvatures do not vary greatly. In case there is a great variance in the curvatures, triangulation is inaccurate and will produce too many errors and fail to find the closest edges of intersecting planes. This method is inapplicable in the case of arbitrary surfaces, with too many problematic vertices occurring that require manual reconnection (Fig. 5).

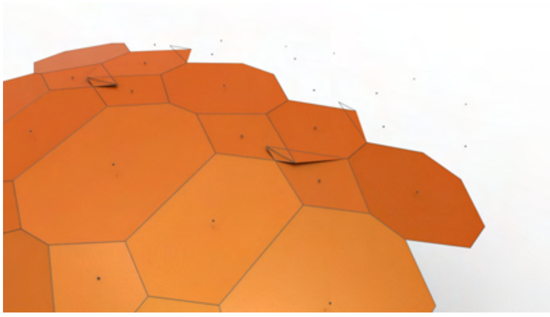

Fig. 5 Tangent plane intersection with problems at some locations

A possible way to take a double curved surface which can be used for a correct triangulation is the sphere. Because of its constant curvature behavior a triangulation of arbitrarily distributed points on a sphere can be achieved by using either the arc length on the great circles which serve as geodesics on the surface or the angles of the centre. Both methods can be employed as utilities measuring the distance between two different surface points A and B and to perform a spatial triangulation (see Fig. 6). Another possibility to get correct triangulations on a double curved surface is the use of a paraboloid of revolution.

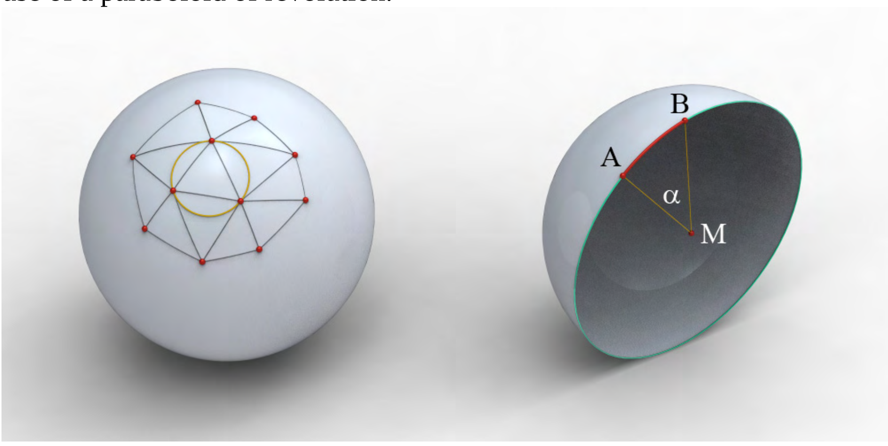

Fig. 6 Triangulation on a sphere (left). Distances between two points A and B can be measured by the associated great circle or the centre angle α.

Affected by the situation on the sphere, we developed the following new idea to construct an appropriate triangulation of our point set P_i to get a correct intersection algorithm. For our approach we consider the osculating paraboloid Π of a point P on a surface S [Kruppa 1955, Pottmann et al. 2007]. The surface Π reflects the curvature behavior of S at P. Depending on the surface point P (elliptic, hyperbolic or parabolic), the surface Π is an elliptic or hyperbolic paraboloid or a parabolic cylinder. In general, an arbitrary surface point P has two different principle curvatures κ_1 and κ_2 and the associated orthogonal principle directions t_1 and t_2 (see Fig. 7 left). For our procedure we perform a linear geometric transformation T on the surface (and the point set) to gain the following situation in P (Fig. 6 right):

$$|\kappa_1| = |\kappa_2|$$

For this transformation T we use an orthogonal perspective affinity with a fixed plane Φ determined by the point P, its surface normal n_P and one of the principle directions t_i. The second direction serves as the direction of the affinity rays. The transformation factor f can easily be derived by the equation of the osculating paraboloid Π:

$$\Pi\colon z = ½ \cdot \kappa_1 x^2 + ½ \cdot \kappa_1 y^2$$

If we keep the principle curvature κ_1 fixed and change κ_2, we get the factor

$$f = sqrt(\kappa_1/\kappa_2)$$

In the converse case we get

$$f = sqrt(\kappa_2/\kappa_1)$$

The transformation T is nothing but an orthogonal non-uniform 3D-scaling. After that an elliptic point P on S becomes an umbilical point so that a certain area around P “looks like a sphere”. Thus, we can perform a very precise triangulation in the neighborhood of P similar to a real sphere. At the same time, the paraboloid Π is transformed into a paraboloid of revolution.

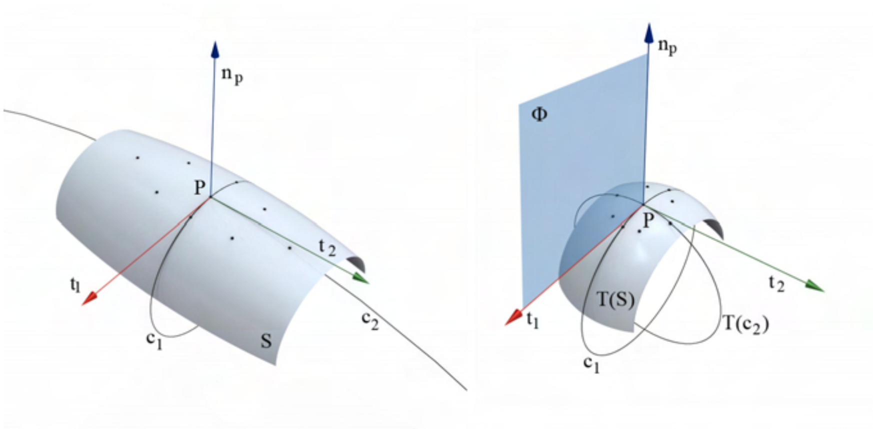

Fig. 7 A surface point P with an adapted coordinate system consisting of the principle directions t_1, t_2 and the surface normal n_P (left). The situation after the transformation T with the fixed plane Φ and the transformed circle of curvature $T(c_2)$ (right). P becomes an umbilical point.

After the transformation T the area around the hyperbolic points "looks like an orthogonal hyperbolic paraboloid" and this can also be used for our procedure, which will be shown later.

The point set P_i is also transformed into a set $T(P_i) = Q_i$ on $T(S)$, which can now be used for the triangulation. The triangulation can be performed in space or by an ordinary 2D-Delaunay triangulation after a projection of the set Q_i into the tangent plane τ_P (Fig. 8). Thus, we get a perfect correlation between the points and know which tangent planes have to be intersected.

The transformation T has to be accomplished for every point P of our set so we get a set T_i , $(i = 1,\ldots,n)$ of transformations. In order to simplify and shorten the whole procedure and to make it feasible we perform a triangulation of the whole set P_i in the [uv]-parameter plane before we achieve the transformations T_i. From this first (incorrect) result we take only a certain set of points in the neighborhood of each point P_i to perform the transformation T_i.

It must be said that this construction only works in a certain neighborhood of the point P and because of the discrete amount of points.

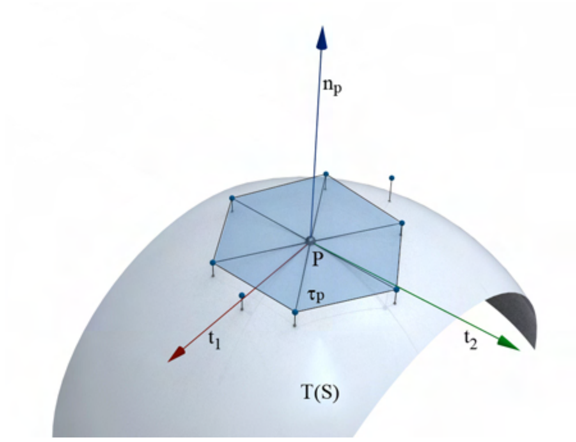

Fig. 8 The correct correlation between the projected and transformed points Q_i

There is one fact which we have to mention concerning the hyperbolic case. If we draw a geodesic circle g round the center *P* of an orthogonal paraboloid (Fig. 8 left) and project it into the tangent plane τ_P, we get a plane curve *g'*. If the "radius" r of c is not that big, the projection g' does not differ much from a circle *c* in τ_P with the same radius r and center *P* (Fig. 9 right). This is the reason why a triangulation of the projected points Q_i in τ_P yields very useful results for our tangent plane intersection algorithm. Again, this construction works only in a certain neighborhood of the point *P*; but that is exactly where our interest lies.

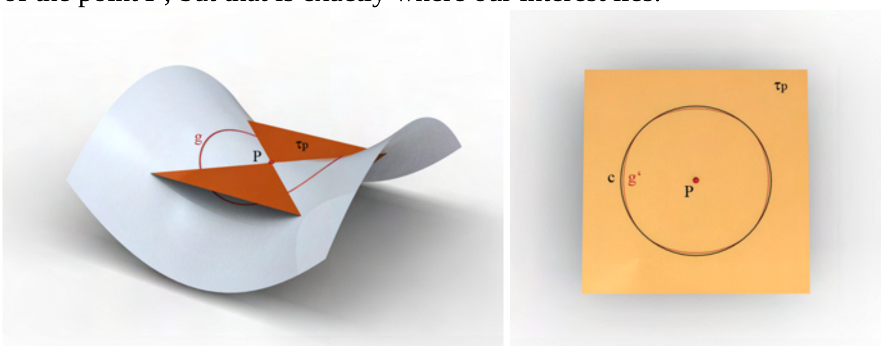

Fig. 9 A geodesic circle *g* with radius *r* round the vertex *P* of an orthogonal hyperbolic paraboloid (left). Its orthogonal projection *g'* into the tangent plane τ_P of *P* does not differ much from a circle *c* with the same radius and the same center *P* (right).

4. Constructing spatial ornaments

In order to construct a spatial ornament consisting of plane elements over a complete arbitrary double curved surface *S* we start with the 2D situation. We take a desired pattern which is constructed as a flat Voronoi diagram to a set of points *SP* and put it over the [*uv*]-parameter plane associated with *S* (Fig. 10 left). Then we map *SP* according to the parameter values onto *S* and determine the spatial triangulation (Fig. 10 right). This yields a first result for the correlation between the points of *SP*.

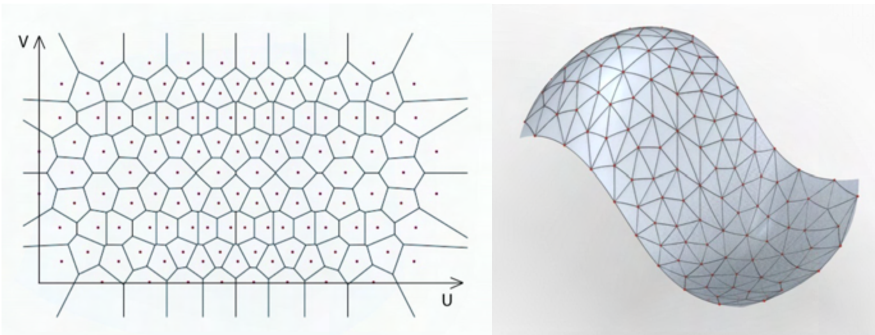

Fig. 10 A plane ornamental pattern designed as a Voronoi diagram to a certain set of points *SP* over the [uv]-parameter plane (left). Mapping of the point set *SP* onto a double curved surface *S* and its spatial triangulation (right).

Now we determine the tangent planes of the set *SP* related to *S* and perform the construction algorithm explained in chapter 3 for all of the points. The geometric transformation *T* needs to be accomplished for very point and its neighbors. The result can be seen in Fig. 11.

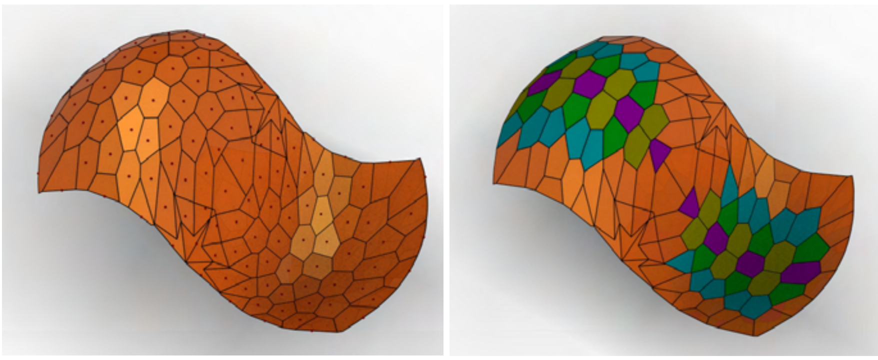

Fig. 11 The desired spatial version of the ornament shown in Fig. 10 for the complex surface *S*. Some of the polygons are colored to enable a comparison with Fig. 10

Additionally Fig. 11 right shows some of the desired polygons of the planar ornament of Fig. 10 in different colors.

The algorithm works quite fine in the positively curved regions of the surface *S*. In the "middle of S" there is a part with parabolic and hyperbolic points. There the spatial ornament differs (of course) from the flat version according to the explanations in chapter 3. Nevertheless our algorithm yields useable panels in a concave shape.

If we use curvature lines instead of the conventional uv - parameter lines we get a different mapping of our point set *SP* onto the surface *S* and therefore a different result. Fig. 12 left shows *S* with a set of its curvature lines and Fig.12 right demonstrates the result of our planarization algorithm. One can see clearly the problems in the "middle of *S*" where we had to put in triangles to close the form. It turned out that for the shape of *S* curvature lines are not an appropriate choice. But changing the parameterization and the parameter lines of a surface would be a good possibility to vary the design. This is a topic which has to be investigated in future.

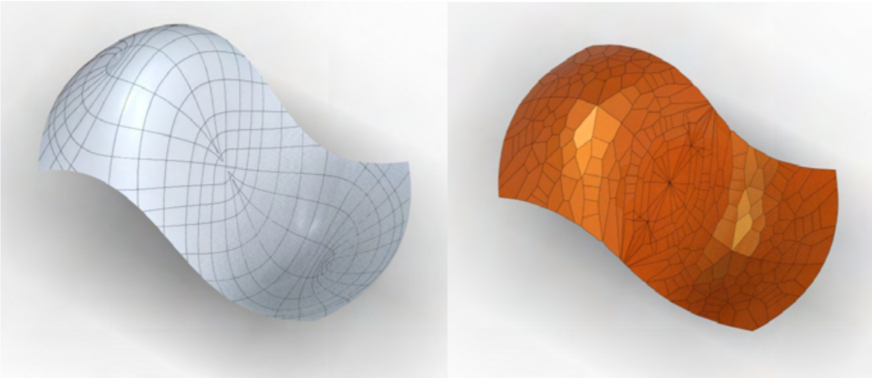

Fig. 12 The surface S and a set of its curvature lines (left). The associated planarization with a problematic region in "the middle" of the surface (right)

In the former chapters we described geometrical principles how to handle with all kind of surfaces in order to analyze the geometrical properties. Especially in figures 10 – 12 a strong curvature behavior can be seen. In the majority of cases architectural design shapes do not have such extreme curvature characteristics. Figure 13 shows an architectural example with a smooth curvature and the implementation of our planar discretization.

Fig 13 A smooth double curved surface segmented in plane façade elements

5. Future work

In the future the great number of ornaments will be analyzed from the aspect of potential transposition to double curved surfaces. Acceptable types of ornaments will be investigated in relation to the type of curvature – positively or negatively curved surfaces. One goal shall be the study of different surface parameterizations and their associated parameter lines. Besides the conventional parameter and curvature lines also conjugate surface lines will we studied.

Other contributions will focus on how a free-form surface can be modified according to a pre-selected ornament so that the type, shape and size of the ornament are preserved. This approach requires the optimization of the surface in relation to a specific criterion of the selected ornament. The issue here is to what extent and according to what rules it is necessary to modify the form of the existing surface so the originally selected ornament lies on it perfectly. Another problem to be investigated is how to limit the infinite range of possibilities to generate a preferred spatial ornament and what conditions can render surface tessellation ornamental in character.

One other major goal will be the implementation of a user interface for the design of a desired ornament including size and shape optimization and first static estimation for standard building materials. This is already an ongoing work. Finally our work should be a device for any designer who wants to approximate and design a double curved surface by means of plane elements according to his aesthetic needs.

The main aim of our whole research project is the implementation of the presented geometric methods through building a self supporting free-form object made out of CLT panels. Figure 14 shows a scale model study there from. Along with the presented geometry topics, associated static conditions and calculations are currently explored in order to make geometric results buildable. This will be discussed in further papers.

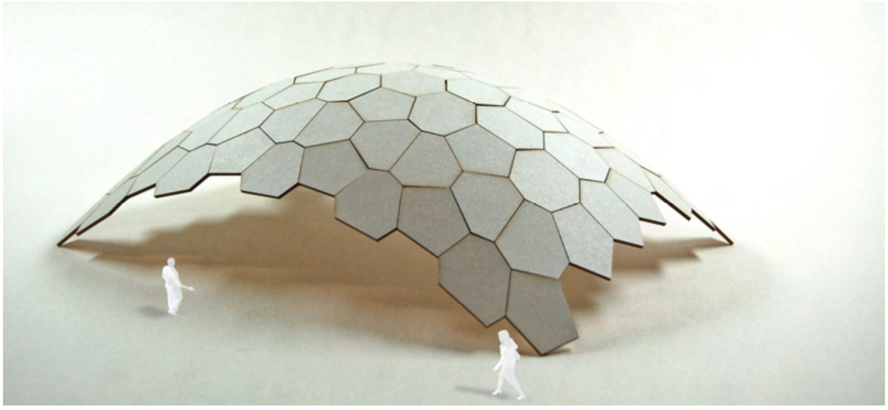

Fig.14 Scale model of an ornamental shell structure

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