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ORNAMENTAL DISCRETISATION OF FREE-FORM SURFACES

Developing digital tools to integrate design rationalisation with the form finding process

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Abstract. The adoption of digital planning methods has given rise to an unprecedented formal freedom in architectural design. Free-form shapes enjoy considerable popularity in architectural production today. However, these shapes prove to be notoriously hard to fabricate. This paper reports on an ongoing research project investigating the approximation of continuous double-curved surfaces by discrete meshes consisting solely of planar facets, which can be constructed efficiently by using standardised, mass-produced building materials. We introduce our geometrical approach, which is based on the intersection of tangent planes to the surface, and present the digital tools we conceived to integrate the processes of design rationalisation and form-finding.

Keywords. Digital tool-making; parametric design; free-form surfaces; design rationalisation; planar discretisation.

1. Introduction

With Leon Battista Alberti's claim that architecture ought to be separate from construction, a profound change of the profession commenced. Being more and more emancipated from the act of building, architects became to be foremost the authors of planning documents, while the problem of realising their designs became the task of engineers and construction specialists. This division of design and production holds up until this day and, moreover, was reinforced by the emergence of digital planning methods during the past decades.

It's just natural that many architects, as authors of design documents, decided to adopt software programs excelling at the production of representational imagery - software which was originally conceived for computer animation or the automotive and aviation industries. These programs enabled architects to create and manipulate even the most complex of shapes and allowed for an unprecedented formal freedom in architectural design.

The most prominent novelty the digital modelling programs brought with them was the introduction of free-form surfaces to the formal language of architectural design. These surfaces were hard to describe using ruler and compass, but could easily be mastered with the aid of a 3d modelling program. However, the software packages did not come with the functionality it takes to rationalise designs so that the new formal vocabulary could be materialised with reasonable efforts.

Design rationalisation can be defined as "the process of approximating an intended form with a well-defined generative principle in order to facilitate building execution" (Fischer 2007, p. 45). Today this task is often performed by digital production specialists after a design has already been conceived. The fruitful process of trying to bring form, structure and the means of their production into the best possible accordance cannot happen in this division of labour.

In this paper we introduce a method to transform continuous double-curved surfaces into discrete meshes consisting solely of planar facets and present the digital tools we conceived to integrate the processes of design rationalisation and form-finding.

2. Discretisation of free-form surfaces

Continuous double-curved shapes have proven to be notoriously hard to fabricate in an architectural scale. Manufacturing free-form building elements usually requires the production of custom moulds for every single building element, which naturally results in high construction costs. For this reason various techniques to adapt the shape of free-form surfaces have been explored to make their construction more efficient (Shelden 2002, Pottmann et al. 2008a, Eigensatz et al. 2010). The approximation of continuous double-curved surfaces by discrete planar meshes has thereby been a topic of great interest, since the fabrication of shapes consisting solely of flat panels is far less costly than the production of curved building elements.

The most straightforward way to transform a continuous surface into flat panels is to approximate the surface with a triangle mesh. This method has been used in a number of outstanding built projects like the Great Court of the British Museum (Norman Foster and Partners with Büro Happold, Figure 1a) or the Fiero Milano (Studio Fuksas with Schlaich, Bergermann and Partners, Figure 1b).

Any arbitrary surface can be triangulated, but when it comes to production the resulting shape is economically less advantageous than equivalent surface structures consisting of panels with more than three sides (Glymph et al. 2004, Pottmann et al. 2008b). Working on the realisation of Frank Gehry's design for the Museum of Tolerance in Jerusalem, Glymph et al. developed a method to tessellate surfaces into planar quadrilateral facets. Their approach is based on the special geometrical properties of translational surfaces (surfaces that are created by sweeping one curve along another curve) and rotational surfaces (created by revolving a curve around an axis) and thus its application is restricted to these classes of shapes. While the Museum of Tolerance was never realised, the roof of Berlin Zoo's Hippopotamus House (Jörg Gribl with Schlaich Bergermann and Partners, Figure 1c) is a built example of a quadrilateral mesh based on a translational surface.

Liu et al. (2006) later introduced a method to discretise a general double-curved surface by optimising a quad mesh that approximates a double-curved shape so that its faces become planar. Similarly, Wang and Liu (2010) developed an approach based on mesh optimisation to produce hexagonal panelisations.

Figure 1. (a) Great Court of the British Museum, (b) Fiero Milano, (c) Berlin Zoo Hippopotamus House.

Our approach to the discretisation of free-form surfaces is based on the intersection of tangent planes (Cutler and Whiting 2007, Troche 2008, Bagger 2010, Stavric and Wiltsche 2011). We take a number of two-dimensional points that can be arbitrarily distributed and map them onto the [uv]-parameter space of a given non-periodic, double-curved surface. We will call the resulting points on the surface "*pattern points*" since they determine the pattern scheme of the emerging planar facets. To every pattern point we determine its tangent plane T to the given surface and intersect it with the tangent planes of adjacent points. As illustrated in Figure 2 the intersection of T with the tangent planes of two adjacent pattern points yields a vertex of the polygon that defines the boundary of the facet laying in tangent plane *T*. Because all vertices found by intersecting with *T* necessarily lay in this plane, the resulting polygon boundary is guaranteed to be planar.

- P....Pattern points (base points of tangent planes)
- T....Tangent planes
- V....Vertex of a panel boundary polygon

Figure 2. Tangent plane intersection.

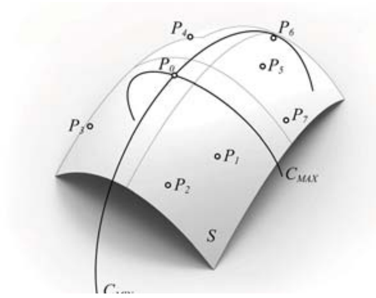
Besides generating absolutely planar facets, a mesh produced by the presented method will naturally tend to a configuration where only three panels meet at each vertex. In comparison, an average of six panels meet at the vertices of triangulated meshes, which leads to far more complicated construction details.

Unlike other discretisation techniques, such as triangulation or quad meshing, shapes created with the presented approach can be composed of polygons with varying numbers of sides. Tangent plane intersections produce a complex ornamental pattern, potentially introducing a new expressive quality to a design project. One of the favourable characteristics of this approach is the level of detailed control it offers to designers. The pattern scheme can easily be altered by simply manipulating the pattern points that determine the tangent planes along the surface. The presented method not only allows to control the discretisation process as a whole, but also enables designers to perform selective optimisations to the discrete shape. Pattern points can be moved, added or deleted locally, without changing the overall geometry. This allows the precise control of the emerging discrete shape - of both aesthetic and performative aspects.

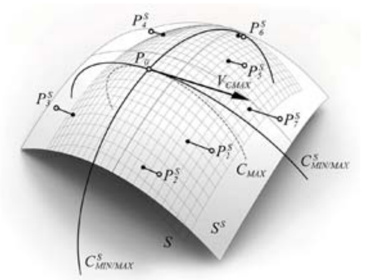
While the calculation of tangent planes to a free-form surface and their intersections is trivial, the challenge is to judge whether two pattern points are adjacent and the intersection of their tangent planes yields a valid edge of a facet boundary polygon. Therefore the interplay of pattern point positions and local surface curvatures must be accounted for.

We have developed an efficient algorithm that solves this problem for surfaces with positive Gaussian curvature and are currently in the process of adapting the method so that it will also be possible to discretise surfaces with negative Gaussian curvature using the same strategy.

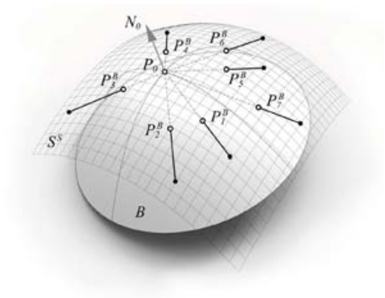
We begin by performing a Delaunay triangulation of the original two-dimensional pattern points. This initial triangulation gives a first result for the correlation between the given points and is used to determine a set of adjacent candidates for every point. To determine which candidates are truly adjacent to a pattern point, the candidate points have to be projected onto the tangent plane of the pattern point that is investigated. We found that if the minimum and maximum curvatures of the surface at the investigated point are equal (which, for example, is the case at the centre of a paraboloid of revolution), the Delaunay triangulation of the projected candidates in the tangent plane reveals the valid set of adjacent points.



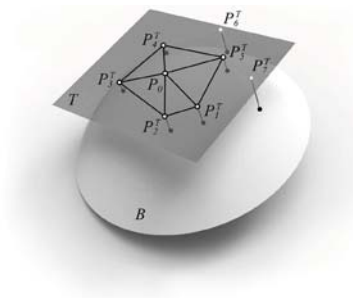
(a) Original pattern points and surface



(b) Scale transformation



(c) Projection onto the osculating paraboloid in P_0



(d) Normal projection onto the tangent plane of P_0 and Delaunay triangulation

Figure 3. Finding the correct set of adjacent points for the tangent plane intersection.

But since surface parts with equal minimum and maximum curvatures are a rare case with free-form shapes, another step is necessary. To compensate for the local surface curvatures, we consider the osculating paraboloid (Pottmann et al. 2007) to the surface S at the investigated point P_0. We calculate the ratio of minimum and maximum surface curvature at P_0 (illustrated as the principal curvature circles C_{MIN} and C_{MAX} in Figure 3b) and scale the candidates according to the square root of this ratio non-uniformly in the direction of the maximum curvature vector V_{CMAX} (Figure 3b). We take the resulting points P_S and project them onto the osculating paraboloid B at P_0 with P_0 as the projection centre (Figure 3c). These transformations compensate the local surface curvature variations around P_0. The resulting points P_B are projected normal onto the tangent plane T in P_0 (Figure 3d). The Delaunay triangulation of the projected points in T (Figure 3d) finally yields the correct correlation between the pattern points: every candidate that is directly connected to P_0 in the triangulation is an adjacent point and its tangent plane must be intersected with the tangent plane of P_0 to get the valid facet laying in the latter plane.

Since thoroughly discussing the geometrical backgrounds of this approach is out of the scope of this paper, I would like to refer the reader to Stavric and Wiltsche (2011), where the geometry of the presented method is discussed in much greater detail. The described process of compensating local surface curvatures has to be repeated for every pattern point. Although this might sound like a time-consuming task, the computations are straightforward. To give a few numbers, the discretisation of a surface consisting of 1000 facets currently takes about 70 milliseconds on a PC equipped with a 3.4 Ghz i-7 quadcore processor. This gave us the opportunity to develop design tools that provide immediate feedback to the actions of a designer, which we deem as an essential feature of our work.

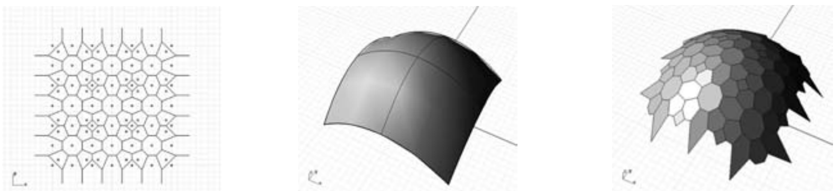
3. The Tools

In the course of our research we have developed a set of digital tools to integrate the presented approach of free-form surface rationalisation into the design process form early design stages on.

The implementation of our tool kit can be divided into two components: the core algorithm (the encoded geometrical and mathematical knowledge to perform the discretisation of a given surface) and the interface provided to control and refine the panelisation process (the actual tools to be used by the designer). We decided to prototypically implement our tools for McNeel's Rhinoceros Software since "Rhino" is excellent with modelling free-form surfaces and enjoys considerable popularity in the architectural community today. Furthermore, due to the common ground of software development for Rhino and its parametric modelling extension, Grasshopper, we were able to efficiently develop explicit modelling tools for the direct manipulation of geometries, as well as a pure parametric modelling interface to our algorithms - both enabling an inherently different workflow.

4. Direct modelling interface

The central element of our direct modelling interface is the "*discrete surface*" - a new constraint-based geometrical object type we introduce to the modelling environment, complementing Rhino's native NURBS surface and polygon mesh objects. The discrete surface is not just a descriptive geometrical object but an active representation that reacts to modifications performed by the designer, automatically forming the panelised approximation of a continuous surface using a given pattern scheme. To create a discrete surface the designer has to define an initial pattern scheme as a set of planar points as well as a double-curved surface to discretise. The pattern scheme points in Figure 4a were created using a small tool we made for the quick generation of panelisation patterns. As depicted, the tool also displays the Voronoi diagram for the generated set of points. Although the emerging panel scheme of the discrete surface deviates from this diagram, it generally gives a good idea what to expect on a surface with positive Gaussian curvature.



(a) Initial pattern scheme *(b) Continuous surface* *(c) Generated discrete surface*

Figure 4. Creating a discrete surface.

Once created, the discrete surface can be refined by simply moving the pattern points along the surface (Figure 5) and by adding new points or removing existing ones. Manipulating pattern points works just like transforming any other points in Rhino, with the exception that they are constrained to move along the surface. While editing the discrete surface, designers can take advantage of the full range of geometrical modelling aids Rhino offers (simultaneous transformation of multiple objects, object snaps, construction grid, etc.).

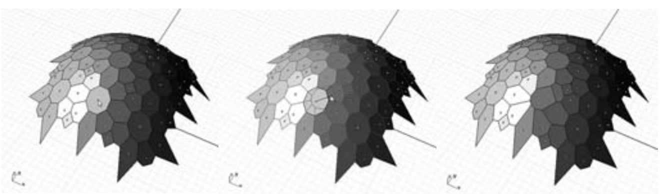


Figure 5. Editing the panelisation scheme by moving pattern points along the surface.

As illustrated in Figure 6, the shape of the original continuous surface can still be transformed by editing the control points of the surface subobject.

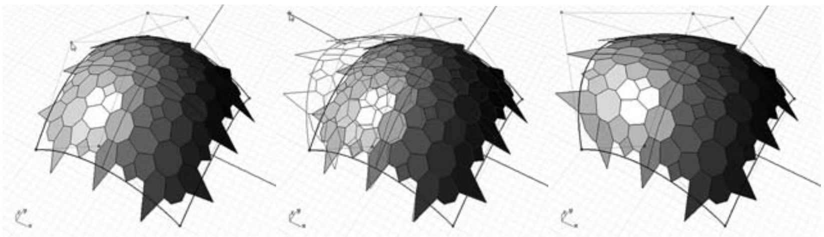


Figure 6. Changing the shape of the continuous surface subobject by editing its control points.

Due to the short computation times of the discretisation algorithm, changes to the surface shape and pattern scheme are reflected in real time, which gives designers immediate feedback on their actions - even when working on a shape consisting of thousands of panels. By providing an active geometrical representation of a discrete surface which allows the interactive refinement of designs, we aim to facilitate an explorative design approach and the continual "talk-back" (Schön 1983) with the design problem. However, relying solely on geometrical constraints is not sufficient in many cases. Therefore we extend our tool kit to Rhino's parametric design environment, Grasshopper. Offering a parametric interface to our algorithm, we enable designers to include their own project specific constraints and to potentially enhance their design approach.

5. Parametric interface

Although the direct manipulation of geometries using explicit modelling tools is generally perceived to be the most intuitive way of designing with digital tools, algorithmic form-finding techniques are employed by a growing number of digitally advanced practitioners. These techniques require that a design is described by a parametric model instead of a concrete geometric representation. The parametric model thereby describes the process how to arrive at a certain form rather than the form itself.

Additionally to the direct modelling interface we created a Grasshopper component that provides a parametric interface to our algorithm. The component can take either geometrically defined or parametrically generated surfaces and pattern point sets as inputs, it outputs the generated discrete mesh geometry which can then be further processed inside Grasshopper.

Figure 7 shows a fully parametric 3d model of a prototype design to be constructed later this year. The structure will be fabricated from cross-laminated timber (CLT) panels and features a custom joint system which was developed for this self-supporting prototype structure (Schimek et al. 2010). The panel geometry, the joints and slots were generated parametrically, based on the output of our Grasshopper component.

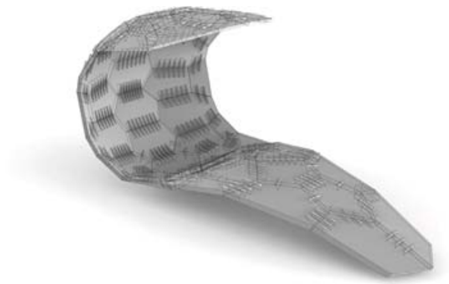


Figure 7. Parametric model of a prototype structure.

6. Outlook

Our future work will focus on three topics. As already mentioned we are currently in the process of adapting our geometrical approach so that it will also be possible to discretise surfaces with negative Gaussian curvature using the same strategy. Our second line of research focuses on the construction of the self-supporting prototype structure mentioned before. We are currently extending our set of tools to incorporate design constraints tailored for this construction strategy. And finally, we are working on the automated generation of data to drive the digital production of panels using computer numerically controlled mills. This will complement our tool kit to form a complete digital chain for the construction of discrete surfaces, reaching from early design stages on through to the fabrication process.

Acknowledgements

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