Discretization of free-form surfaces by plane elements derived from tangent planes

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Keywords

1= discretization 2= free-form surfaces 3= ornament

Abstract

In the last few years, the discretization of free-form surfaces has been a topic of great interest in the field of geometrizing architectural design, representing the first step in the creation of buildable free forms in architecture. This paper presents a new approach to discretize free-form surfaces, with the goal of generating plane panels according to specific aesthetic criteria. Our method is based on the application of tangent planes and their intersection on an arbitrary double-curved surface.

The novel process in this work is that we take plane ornaments and the surface curvature at local points into account. The latter solves former problems which occurred when intersecting the planes. The fact that there is an infinite number of possibilities when selecting tangent planes on a surface raises the issue of the way and conditions which make it possible to select specific tangent planes whose intersection would produce a desired 3D ornamental shape deduced from a 2D ornament.

Introduction

With the digital methods architectural design has become highly complex [Farin 2002, Kolarevic 2003, 2008] and too often the technical development of the construction industry today does not allow the actualization of non-standard architectural design, which is characterized by higher degrees of geometrical complexity. Finding the right measure in terms of complexity and appropriateness of digital design and the possibility to make the construction of such structures efficient and feasible represents a highly complex set of issues that must be included at the beginning of the architectural design process.

The problem characterizing the actualization of new non-standard forms today is the lack of standards and norms in terms of their construction using traditional building materials, such as concrete, wood glass or steel. The currently used norms have standardized those structural forms and systems as confirmed by experiments and practice, and any structure departing from these norms in terms of structural properties represents a challenge and asks for a new approach to problem solving. Interdisciplinary research combining the efforts of architects, structural engineers and building material technologists is needed to define new standards, classify forms and shapes, analyze the impact and action of stress, optimize forms and structures and examine connections between various components.

The first step in the creation of buildable free forms in architecture is their discretization. The discretization of free-form surfaces is defined as the process of partitioning a continuous surface into discrete counterparts that make a surface suitable for numerical evaluation and calculation. Free-form surfaces may be discretized in a number of ways and in accordance with a number of principles. One method of surface discretization results in planar panels - panelization, whereas with another curved segments are obtained. In terms of geometry, there are two fundamental differences between these two methods of discretization. First, when a surface is discretized into planar elements, the initial surface form is approximated in order to obtain planar elements, and depending on the size of individual elements, a greater or lesser distortion in the geometry of the set form occurs. With the second method of discretization, the set form is not approximated; instead, the complete form is partitioned into smaller curved segments. When a surface is discretized into planar elements, the obtained segments of the surface share straight boundary edges, while with curved segments the boundary edges are curved.

In general curved structures are much more efficient than flat structures due their ability to activate membrane forces thus being much stiffer [Weber, 2009]. Nevertheless, if a surface has to be discretized in view of its actual construction, obviously from the aspect of technology discretization into planar elements is both more feasible and cost-effective, in comparison with discretization into curved elements [Weber 2009] and today is a very actual topic.

In this work we will show the panelization of free form surfaces with both positiv and negative Gaussian curvature by means of tangent planes

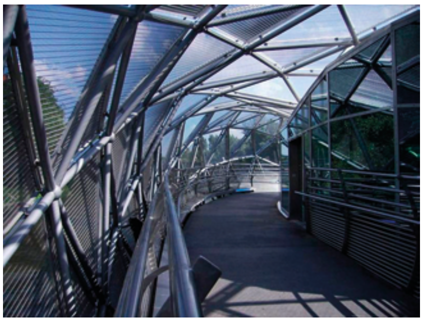

Figure 1

Discretization by triangulation – Murinsel Graz, Austria

and their intersections. The position of the tangent planes we will choose are based on 2D generated patterns. After the pattern transfer onto the 3D surface only minor deviations occur as a consequence of the space transformation. This is possible because we take the surface curvature at local points into account that solves former problems which occurred when intersecting the tangent planes and transfering the desired 2D pattern.

Discretization into plane segments - panelization

A surface can be partitioned into plane elements using different techniques, triangulation, quad meshing, or intersection of freely placed tangent planes. There are both advantages and disadvantages to each of these methods of discretization, which will be discussed briefly.

Triangulation is the best-known method of curved surface discretization (Fig. 1). This method is used for partitioning a selected surface into triangular planar segments. The drawback of this method is a very large number of edges characterized by a high degree of geometric complexity, which in turn requires many load-carrying members, large quantities of structural materials, and increases construction costs. When it comes to the aesthetics, only the size and aspect ratio of triangular panels can be influenced.

The second panelization method is quad meshing, where a surface is divided into plane quadrangular

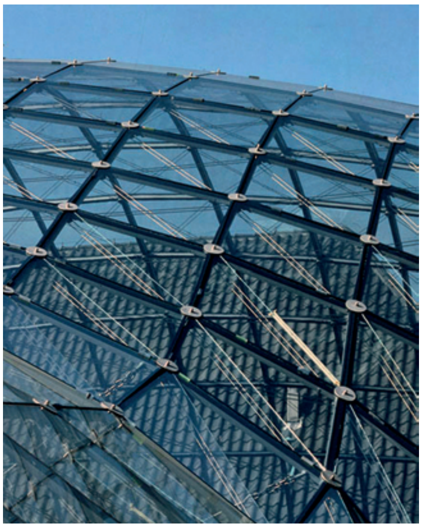

Figure 2

Quadrangular polygons - Museum of Hamburger History, Hamburg

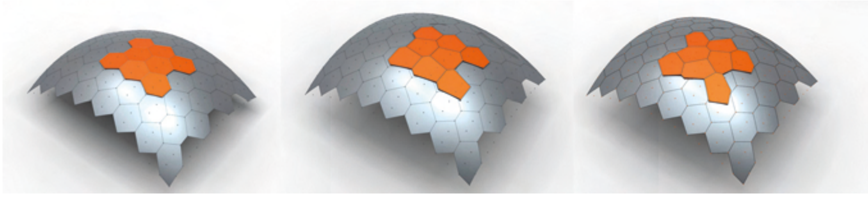

Figure 3

Different choices of tangent planes produce different plane panels

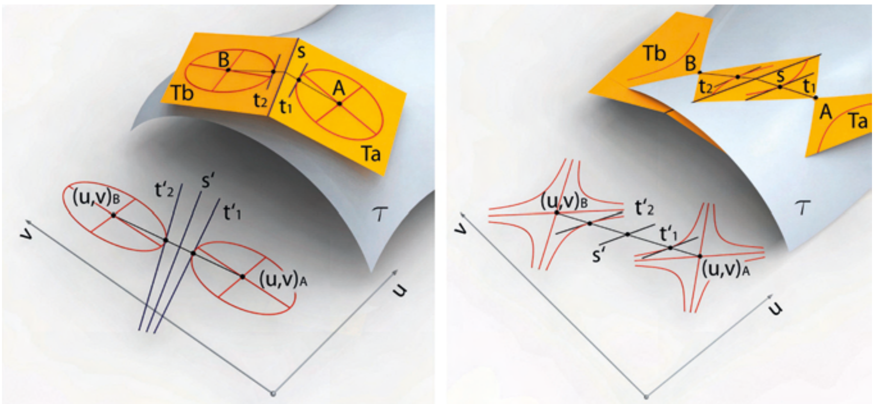

Figure 4.

Consideration of local curvature of surface t (positive curvature – left and negative curvature – right) and transfer to [u,v]-plane.

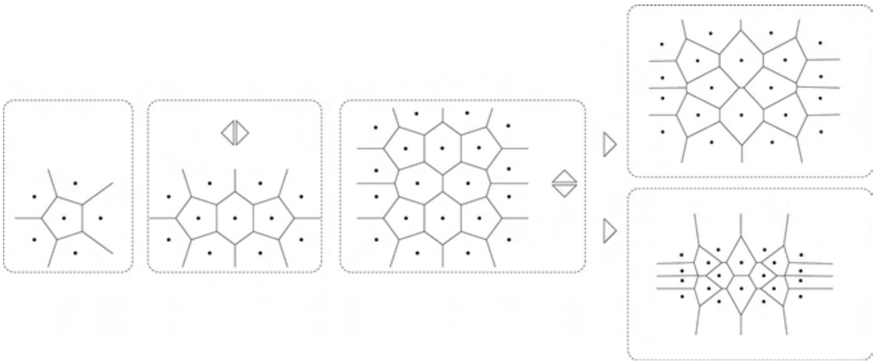

Figure 5

Generation of the ornamental pattern starting with the n-sided pattern-cell on the left by mirroring and moving the cell points in orthogonal symmetry directions. Generation of different ornamental patterns by scaling the cell points in the symmetry directions (rightmost side).

polygons [Sauer 1970, Glymph et al. 2002, Liu et al. 2006, Pottmann et. al. 2008, Zadravec et al. 2010]. From the aspect of use of materials, quad meshing is more optimal than tessellation into triangular elements. However, it cannot be easily employed with arbitrary surfaces.

The third panelization type is a tessellation, where plane polygons of different topology can be obtained. The construction is based on the duality between the surface points and their tangent planes. The tessellation works on the intersection of numerous tangent planes of the selected surface. If the tangent planes of a smooth surface are positioned in a certain way, it is possible to obtain e.g. convex hexagonal "honeycomb" panels. This is achievable if the observed surface is positively curved. This topic is discussed for instance in Wang (2008) and Wang (2009). If the surface is negatively curved, we get butterfly-formed panels. If tangent planes are selected at consecutive points near the principle curvature lines, it is possible to predict the orientation of the intersecting edge between two adjacent surfaces. Following this principle, the paper [Troche, 2008] also uses a TPIAFT algorithm (Tangent Plane Intersection – Advanced Front Technique) to panelize a positively curved surface and optimize the size and shape of the panels, although without applying the aesthetic principle.

In Bagger 2009 [1], the tangent plane method is used for the definition of facetted self-supported shell structures made from glass. This method is analyzed on smooth surfaces with positive Gaussian curvature and six-sided honeycomb tessellation. The work offers guidelines for the design of plate shell structures based on the connection between glass elements. Unlike this work, our paper analyzes different geometrical ornamental patterns with surfaces of both positive and negative curvature. Structural analysis is not taken into consideration.

Discretization with desired pattern

The fact that there is an infinite number of possibilities when selecting points on a surface through which tangent planes can be placed raises the issue of the way and conditions which make it possible to select specific tangent planes whose intersection would produce a desired shape in accordance with the previously selected tessellation, a 3D ornament (Fig. 3). Another issue is whether there is an infinite range of possibilities to generate a preferred 3D ornament and on what conditions surface tessellation would be ornamental in character, i.e. it would produce not only the functional, but also the aesthetic component of a free-form surface.

Our approach to this problem is based on finding a connection between 2D patterns and patterns obtained by selecting the right tangent planes on a surface and their intersections. In order to solve this problem, local surface curvature and Dupin's indicatrices are taken into account (Fig. 4). Two arbitrary points *A* and *B* are selected on a free-form surface t. The points *A* and *B* on t have certain parameters $(uv)_A$ and $(uv)_B$. To *A* and *B* tangents planes T_A and T_B are determined and they define their line of intersection *S*. The direction vector of the intersection line *S* can be estimated by averaging the conjugate directions in *A* and *B* with respect to the geodesic line in between and the help of the associated Dupin's indicatrices. Fig. 4 left shows points *A* and *B* lying in the elliptic part of the surface, so the Dupin's indicatrices are ellipses, while in Figure 4 right points *A* and *B* are in the hyperbolic part of the surface, so the Dupin's indicatrices consist of two pairs of hyperbolae (one pair cannot be seen because it is under the surface t). If the Dupin's indicatrices are transferred onto the *[u,v]*-plane, the direction vector of the intersection line can also be estimated in 2D space. We will use this type of approximation to generate our 2D ornamental patterns. The fundamental property of a pattern generated in this way is that when it is used on a 3D free surface, it preserves the initial shape and form. Minimal pattern deviation occurs due to the transfer of the 2D pattern to the 3D model. This pattern is topologically equal to the Delaunay triangulation used here to determine adjacent intersecting planes.

The further problem concerns the arrangement of the points and their tangent planes on the surface t in order to produce the right intersection lines for the generation of the pattern of the whole surface. We resolve this problem by selecting a single pattern cell whose multiplication and transformation can produce various patterns. Fig. 5 shows a pattern generated in this way. In this case, a pattern is generated for a surface of positive curvature whose Dupin's indicatrices are elliptical. The basic cell in the *[u,v]*-plane consists of *n+1* points arranged in such a way that it is n-sided and symmetrical (Fig. 5 shows pentagonal symmetry). The next step implies the transformation of the points of the basic cell (mirroring, moving, scaling), resulting in different patterns with various polygons. Interestingly, the Voronoi diagram based on the cell points results in a pattern which approximates the final surface pattern with positive Gaussian curvature. Depending on the curvature characteristics the surface pattern deviates from the Voronoi diagram. When it comes to surfaces of positive curvature panels remain convex, whereas concave polygons are created on surfaces of negative curvature.

Figure 6 shows different examples of ornamental cells, 2D patterns, arrangements of points and final surface panelizations. Figures 6(1-4) show surfaces with positive curvature, while Figures 6(5-7) show surfaces with negative curvature. The ornaments generated on surfaces with positive curvature consist of convex polygons, while those obtained on surfaces with negative curvature are composed mainly of concave polygons. The spatial ornaments in figures 6(3), 6(4) and 6(7) show also four panels meeting in one point. This only happens because of a special configuration between the 2D pattern and the surface. In the case of surfaces points with balanced principle curvatures the intersecting configuration of the tangent planes can be taken directly from the 2D Delaunay triangulation of the cell points. Surface points, whose principle curvatures differ strongly, must be treated in a special way taking the local curvature into

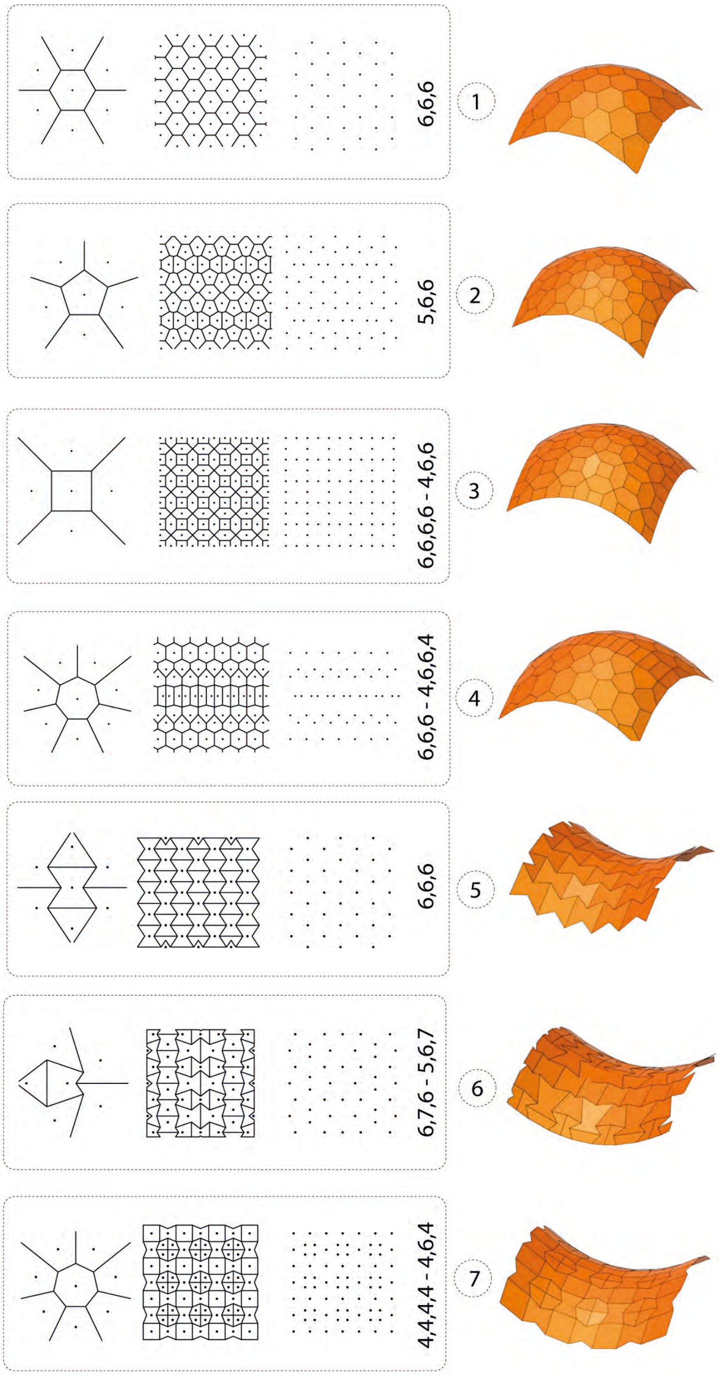

Figure 6

Different panelizations on positive and negative curved surfaces

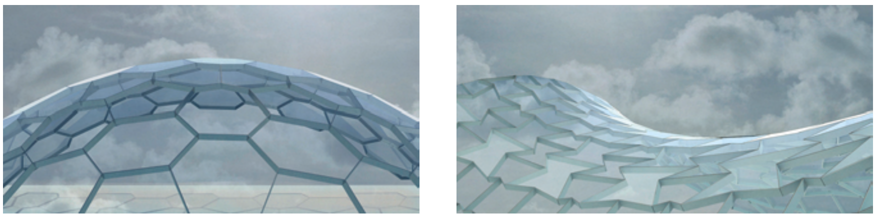

Figure 7 and 8

Structure in glass

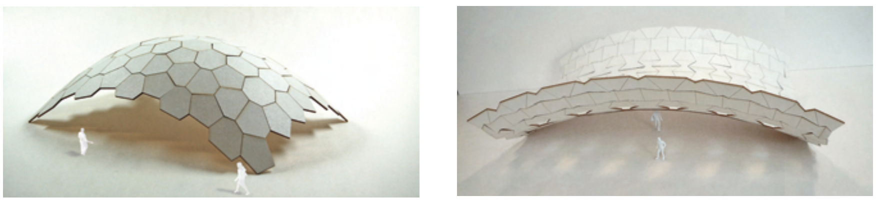

Figure 9 and 10

Scale model of self supported structure account [Stavric, Wiltsche 2011]. 2010

Two factors need to be considered in terms of using glass with this type of structure. The above principles are applicable in shell structures for surfaces with positive curvature and smooth geometry [Bagger 2009], whereas cables must be used to counter tensile stress for surfaces with negative curvature. The second factor concerns the fact that glass is the non-bearing skin element, while it is the steel frame that supports surfaces of arbitrary curvature and shape. Depending on the surface chosen, it is also possible to combine different kinds of support structures for different parts of a complex surface (Fig. 7, 8, 9 and 10).

Conclusions

The development of digital technologies in the last twenty years has led to an unprecedented formal freedom in design and in the representation in virtual space. Combining non-standard geometry with CAD tools enables new ways to express and materialize architectural ideas and concepts. The transformation of a virtual double-curved surface into a buildable physical structure and object is always accompanied by huge costs and major problems, such as geometric and structural ones.

This paper presents a new simple geometric approach to panelizing free-form surfaces, with the goal of ording to specific aesthetic criteria. This analysis is based on previously solving the problem of TPI (tangent plane intersection) on free-form surfaces. Construction also takes into account the curvature properties of the given free-form surface and provides local control. Local control is a great advantage over structures where optimization algorithms are used "to design our design". The main problem in the process of discretization of free-from surfaces is choosing the correct adjacent tangent planes that model polygonal panels and determine the direction vectors of their edges.

Acknowledgements

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