Ornamental Discretisation of Free-form Surfaces

Developing digital tools to integrate design rationalisation with the form finding process

Markus Manahl, Milena Stavric and Albert Wiltsche

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Abstract

The adoption of digital planning methods has given rise to an unprecedented formal freedom in architectural design. Free-form shapes enjoy considerable popularity in architectural production today. However, these shapes prove to be notoriously hard to fabricate. In the course of a funded research project we investigated the approximation of continuous double-curved surfaces by discrete meshes consisting solely of planar facets, which can be fabricated efficiently using standardised, mass-produced building materials. We introduce our geometrical approach, which is based on the intersection of tangent planes to the surface, and present the digital tools we conceived to integrate the processes of design rationalisation and form-finding.

1. INTRODUCTION

With Leon Battista Alberti's claim that architecture ought to be separate from construction, a profound change of the profession commenced. Being more and more emancipated from the act of building, architects became to be foremost the authors of planning documents, while the problem of realising their designs became the task of engineers and construction specialists. This division of design and production holds up until this day and, moreover, was reinforced by the emergence of digital planning methods during the past decades.

It's just natural that many architects, as authors of design documents, decided to use CAD programs that would speed up the traditional process of drawing plans by hand significantly. But the new software – which was for the largest part based on applications originally developed for computer animation or the automotive and aviation industries – would eventually become much more than just a tool to raise efficiency. It enabled architects to create and manipulate even the most complex of shapes with unprecedented ease and thus allowed the effortless exploration of new formal possibilities in virtual space.

One of the most prominent novelties the digital modelling programs brought with them was the introduction of free-form surfaces to the formal language of architectural design. These surfaces were hard to describe using ruler and compass, but could easily be mastered with the aid of a 3d modelling program. However, it is one thing to conceive a virtual free-form surface, but it's a completely different story to then realise such a design in an architectural scale. Current modelling programs typically excel at the production of representational imagery, but they do not come with the functionality it takes to rationalize designs so that the new formal vocabulary can be materialized with reasonable efforts.

Design rationalisation can be characterised as "the process of approximating an intended form with a well-defined generative principle in order to facilitate building execution" [1]. Today this task is often performed by digital production specialists, after a design has already been conceived. The fruitful process of trying to bring form, structure and the means of their production into the best possible accordance cannot happen in this division of labour.

In this paper we discuss a method to transform continuous double-curved surfaces into discrete meshes consisting solely of planar facets and present the digital tools we conceived to integrate the processes of design rationalisation and form-finding.

2. DISCRETISATION OF FREE-FORM SURFACES

Continuous double-curved shapes have proven to be notoriously hard to fabricate in an architectural scale. Manufacturing free-form building elements usually requires the production of custom moulds for every single building element, which naturally results in high construction costs. For this reason various techniques to adapt the shape of free-form surfaces have been explored to make their construction more efficient [2,3,4]. The approximation of continuous double-curved surfaces by discrete planar meshes has thereby been a topic of great interest, since the fabrication of shapes consisting solely of flat panels is far less costly than the production of curved building elements.

The most straightforward way to transform a continuous surface into flat panels is to approximate the surface with a triangle mesh. This method has been used in a number of outstanding built projects like the Great Court of the British Museum (Norman Foster and Partners with Buro Happold, Figure 1a) or the Fiero Milano (Studio Fuksas with Schlaich, Bergermann and Partners, Figure 1b).

Any arbitrary surface can be triangulated, but when it comes to production the resulting shape is economically less advantageous than equivalent surface structures consisting of panels with more than three sides [5,6]. Working on the realisation of Frank Gehry's design for the Museum of Tolerance in Jerusalem, Glymph et al. [5] developed a method to tessellate surfaces into planar quadrilateral facets. Their approach is based on the special geometrical properties of translational surfaces (surfaces that are created by sweeping one curve along another curve) and rotational surfaces (created by revolving a curve around an axis) and thus its application is restricted to these classes of shapes. While the Museum of Tolerance was never realised, the roof of Berlin Zoo's Hippopotamus House (Jörg Gribl with Schlaich Bergermann and Partners, figure 1c) is a built example of a quadrilateral mesh based on a translational surface.

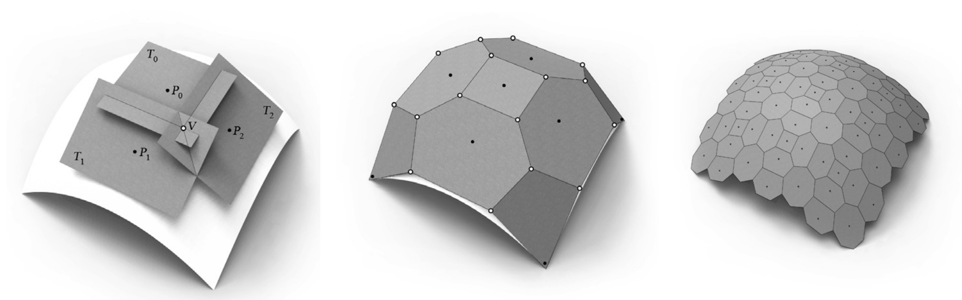
▼ Figure 1: (a) Great Court of the British Museum, (b) Fiero Milano, (c) Berlin Zoo Hippopotamus House

Working with translational and rotational surfaces has a number of advantages. For example, it allows the creation of meshes that have greater similarity between panels, which can help to further reduce construction costs. However, it also imposes a formal limitation to designers and requires a less intuitive design process. Liu et al.[7]. developed a method to discretise general double-curved surfaces employing a mesh optimisation technique. They begin with an initially non-planar quad mesh approximating a free-form shape and then optimise the mesh so that its faces become planar. Similarly, Wang and Liu [8] introduced an approach based on mesh optimisation that allows the creation of hexagonal panelisations.

3. TANGENT PLANE INTERSECTIONS

Our approach to the discretisation of free-form surfaces is based on the intersection of tangent planes [9,10,11,12]. We take a number of two-dimensional points that can be arbitrarily distributed and map them onto the [uv]-parameter space of a given non-periodic, double-curved surface. We will call the resulting points on the surface "*pattern points*" since they determine the pattern scheme of the emerging planar facets. To every pattern point we determine its tangent plane *T* to the given surface and intersect it with the tangent planes of adjacent points. As illustrated in figure 2 the intersection of *T* with the tangent planes of two adjacent pattern points yields a vertex of the polygon that defines the boundary of the facet laying in tangent plane *T*. Because all vertices found by intersecting with *T* necessarily lay in this plane, the resulting polygon boundary is guaranteed to be planar.

► Figure 2: Tangent plane intersection

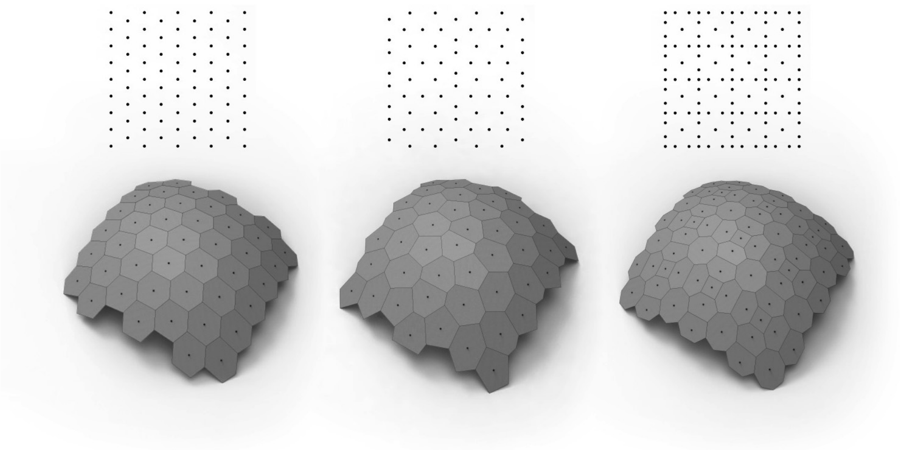


P.... Pattern points (base points of tangent planes)

T.... Tangent planes

V.... Vertex of a panel boundary polygon

Besides generating absolutely planar facets, a mesh produced by the presented method will naturally tend to a configuration where only three panels meet at each vertex (figure 3). In comparison, an average of six panels meet at the vertices of triangulated meshes, which leads to far more complicated construction details.



◀ Figure 3: Different pattern point schemes and resulting discrete meshes

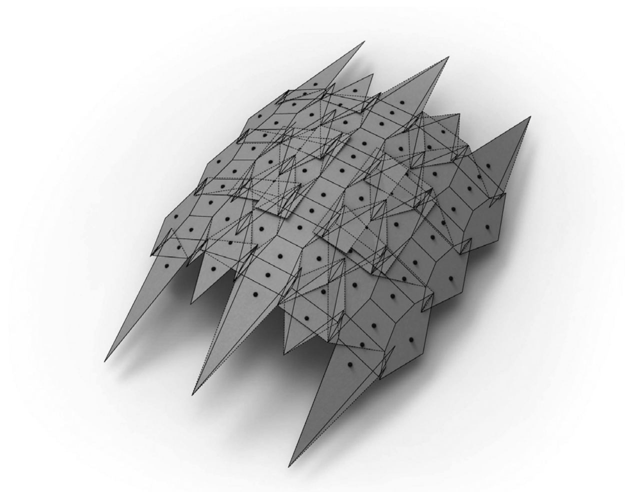
Unlike other discretisation techniques, such as triangulation or quad meshing, shapes created with the presented approach can be composed of polygons with varying numbers of sides. Tangent plane intersections produce a complex ornamental pattern, potentially introducing a new expressive quality to a design project.

One of the favourable characteristics of this approach is the level of detailed control it offers to designers. The pattern scheme can easily be altered by simply moving the pattern points that determine the tangent planes along the surface. The presented method not only allows to control the discretisation process as a whole, but also enables designers to perform selective optimisations to the discrete shape. Pattern points can be translated, added or deleted locally, without changing the overall geometry. This allows the precise control of the emerging discrete shape – of both aesthetic and performative aspects.

3.1. Finding the correct set of adjacent tangent planes to intersect

While the calculation of tangent planes to a free-form surface and their intersections is trivial, the challenge that comes with this method is to judge whether two tangent planes are adjacent and their intersection yields a valid edge of a facet boundary polygon. If we choose the wrong planes to intersect we get self-intersecting polygons, meshes with holes and ultimately shapes that might have an intriguing visual complexity but would again be rather hard to fabricate (figure 4).

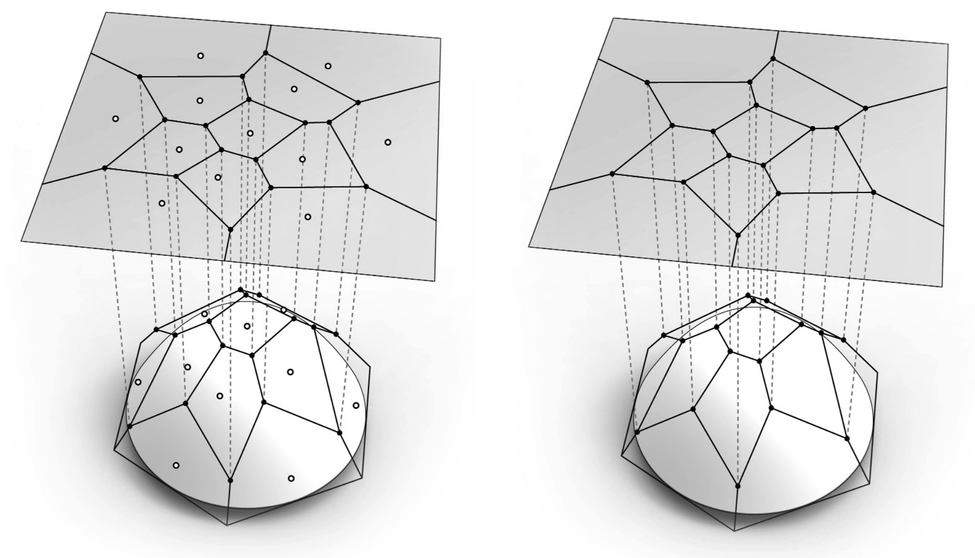
► Figure 4: Choosing the wrong planes to intersect



In the course of our research project we have developed an efficient algorithm that solves the problem of finding the correct set of adjacent tangent planes to intersect for surfaces with positive Gaussian curvature, and we hope we get the chance to adapt the method in an upcoming follow-up project so that it will also be possible to discretise surfaces with negative Gaussian curvature using the same efficient strategy.

To explicate our approach we will first have a look at a regular paraboloid surface. For the discratisation of the paraboloid in figure 5 we chose a completely random set of pattern points. As illustrated on the left side in figure 5 we project the pattern points onto a plane orthogonal to

► Figure 5: Finding the correct set of adjacent tangent planes to intersect



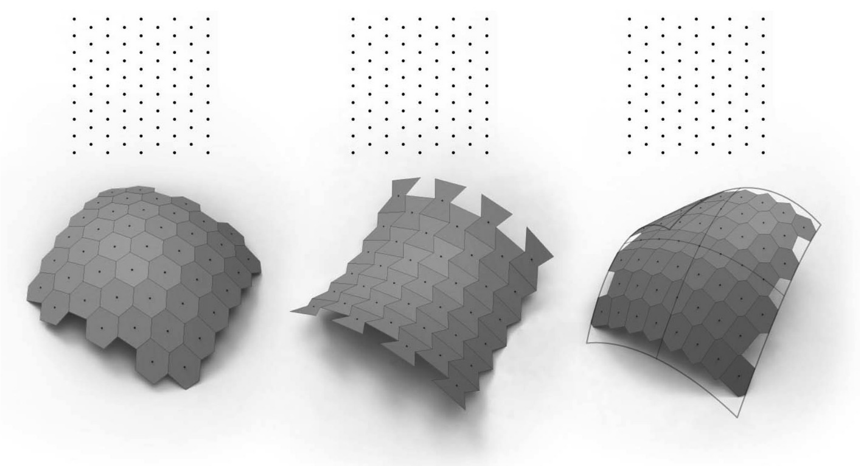
the paraboloid's axis of revolution and create a two-dimensional Voronoi diagram from this set of points. As you can see on the right side, the orthogonal projection of the discrete mesh created by tangent plane intersections exactly matches the Voronoi diagram we created from the projected pattern points [13]. This means, first, that we can work with the pattern points on the surface (which are the points of contact between surface and tangent planes) rather than the tangent planes themselves, and second, that we can use the Voronoi diagram (respectively the Voronoi diagram's dual – the Delaunay triangulation) to determine which tangent planes must be intersected.

It's that simple, at least for paraboloid surfaces. Of course regular paraboloids are a rare case in free-form design, and indeed things get a bit trickier when it comes to free-form surfaces.

3.2. Compensation of local surface curvatures

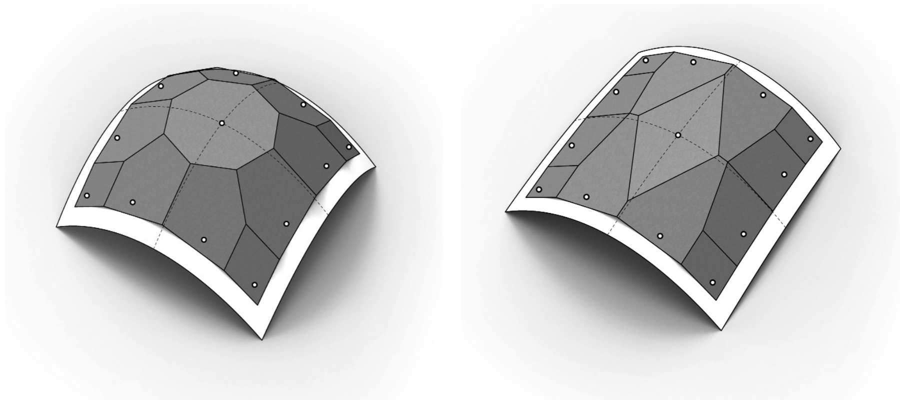
Figure 6 shows three different surfaces with the same initial pattern point scheme. We can see that changing surface curvatures has a certain influence on the emerging panelisation scheme, but generally a pentagon will remain a pentagon and a hexagon will remain a hexagon, even if it turns into a non-convex polygon like on the hyperbolic surface in figure 6. This means, for the examples shown in figure 6 we can intersect the same sets of tangent planes in each case, regardless of surface curvatures.

However, if the difference between minimum and maximum surface curvatures is higher than in the above examples, the polygonal composition of a discrete shape might change, as illustrated in figure 7. Here we have two surfaces which were discretised using the same initial pattern point scheme. On the regular symmetrical surface on the left we get an octagon as the central polygon. The surface on the right side exhibits highly anisotropic curvatures: while it is bent fairly strong in one direction, it



◀ Figure 6: Influence of surface curvatures on the emerging panelisation scheme

▶ Figure 7: Different polygonal compositions due to surface curvature variations

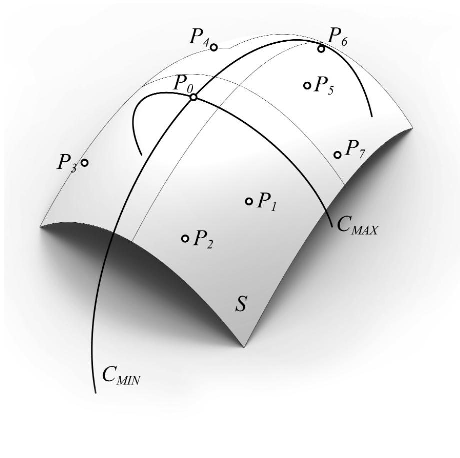


shows almost no curvature in the other. Due to the changing angles of tangent planes relative to each other, the central polygon here turns into a quadrilateral. So we have to intersect the central tangent plane with eight adjacent planes for the left surface in figure 7 and only with four planes in case of the right surface, even though the pattern points scheme is the same for both surfaces. We can conclude that another step is necessary to find the correct set of adjacent tangent planes, since local surface curvatures must also be accounted for.

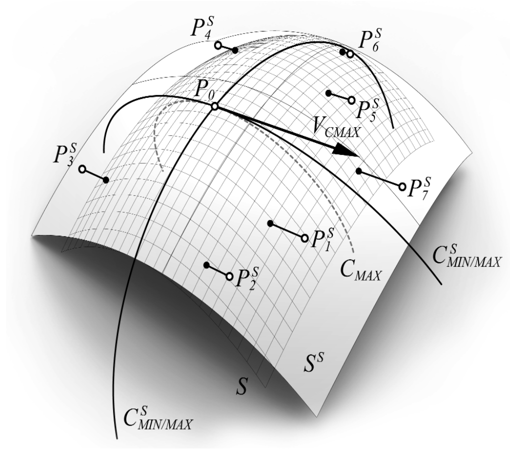
We begin by performing a Delaunay triangulation of the pattern points in the surface's two-dimensional parameter space. This initial triangulation gives a first result for the correlation between the given points and is used to determine a set of adjacent candidates for every point. From here on we investigate every pattern point separately to determine which candidates are truly adjacent and in consequence, which tangent planes must be intersected.

To compensate local surface curvatures we consider the osculating paraboloid [14] to the surface S at the investigated point P_0 (figure 8). We calculate the ratio of minimum and maximum surface curvature at P_0 (illustrated as the principal curvature circles C_{MIN} and C_{MAX} in figure 8b) and scale the adjacent candidates P_1 to P_7 according to the square root of this ratio non-uniformly in the direction of the maximum curvature vector V_{CMAX} (figure 8b). We take the resulting points P_S and project them onto the osculating paraboloid B at P_0 with P_0 as the projection centre (figure 8c). These transformations compensate local surface curvature variations around P_0. The resulting points P_B are projected orthogonally onto the tangent plane T in P_0 (figure 8d). The Delaunay triangulation of the projected points in T (figure 8d) finally yields the correct correlation between the pattern points: every candidate that is directly connected to P_0 in the triangulation is an adjacent point and the corresponding tangent plane must be intersected with the tangent plane of P_0 to get the valid facet laying in the latter plane.

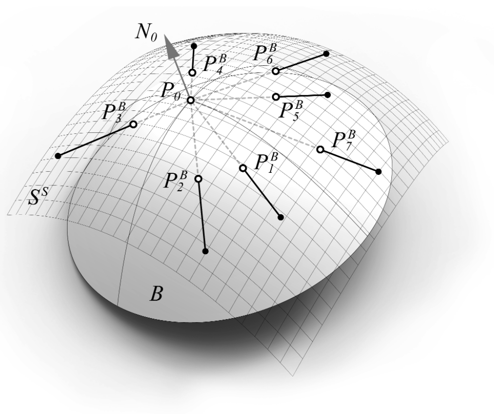
◀ Figure 8: Compensation of surface curvatures



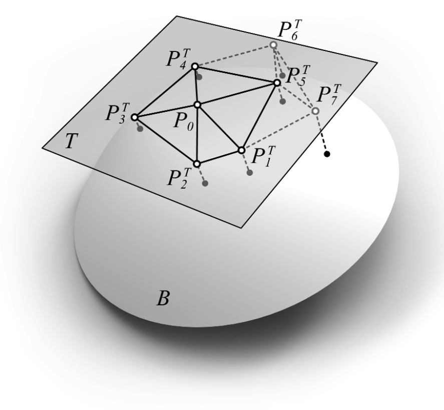
(a) Original pattern points and surface



(b) Scale transformation



(c) Projection onto the osculating paraboloid in P_0



(d) Normal projection onto the tangent plane of P_0 and Delaunay triangulation

The described process of compensating local surface curvatures has to be repeated for every pattern point. Although this might sound like a time-consuming task, the computations are straightforward. To give a few numbers, the discretisation of a surface consisting of 1000 facets currently takes about 70 milliseconds on a PC equipped with a 3.4 Ghz i-7 quadcore processor. This gave us the opportunity to develop design tools that provide immediate feedback to the actions of a designer, which we deem as an essential feature of our work.

4. THE TOOLS

In the course of our research we have developed a set of digital tools to integrate the presented approach of free-form surface rationalisation into the design process from early design stages on.

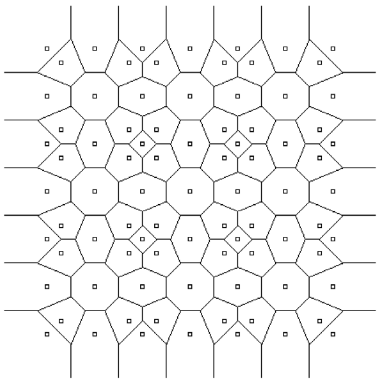
The implementation of our tool kit can be divided into two components: the core algorithm (the encoded geometrical and mathematical knowledge to perform the discretisation of a given surface) and the interface provided to control and refine the panelisation process (the actual tools to be used by the designer). We decided to prototypically implement our tools for McNeel's Rhinoceros 3D software since "Rhino" is excellent with modelling free-form surfaces and enjoys considerable popularity in the architectural community today. Furthermore, due to the common ground of software development for Rhino and its parametric modelling extension, Grasshopper, we were able to efficiently develop explicit modelling tools for the direct manipulation of geometries, as well as a pure parametric modelling interface to our algorithms – both enabling an inherently different workflow.

4.1. Direct modelling interface

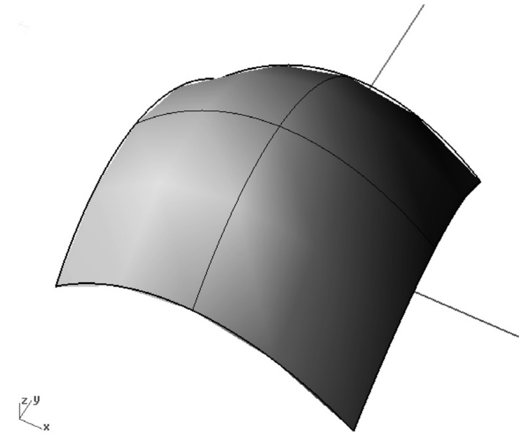
The central element of our direct modelling interface is the *"discrete surface"* - a new constraint-based geometrical object type we introduce to the modelling environment, complementing Rhino's native NURBS surface and polygon mesh objects. The discrete surface is not just a descriptive geometrical object but an active representation that reacts to modifications performed by the designer, automatically forming the panelised approximation of a continuous surface using a given pattern scheme.

To create a discrete surface the designer has to define an initial pattern scheme as a set of planar points as well as a double-curved surface to discretise. The pattern scheme points in figure 9a were created using a small tool we conceived for the quick generation of panelisation patterns. As depicted, the tool also displays the Voronoi diagram for the generated set of points. As we already discussed, the emerging panel scheme of the discrete surface deviates from this diagram, but it generally gives a good idea what to expect on a surface with positive Gaussian curvature.

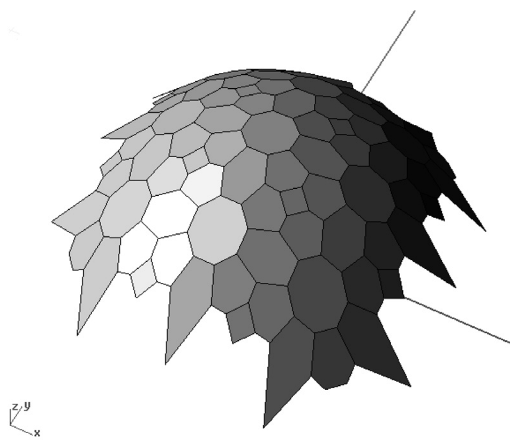
◀ Figure 9: Creating a discrete surface



(a) Initial pattern scheme

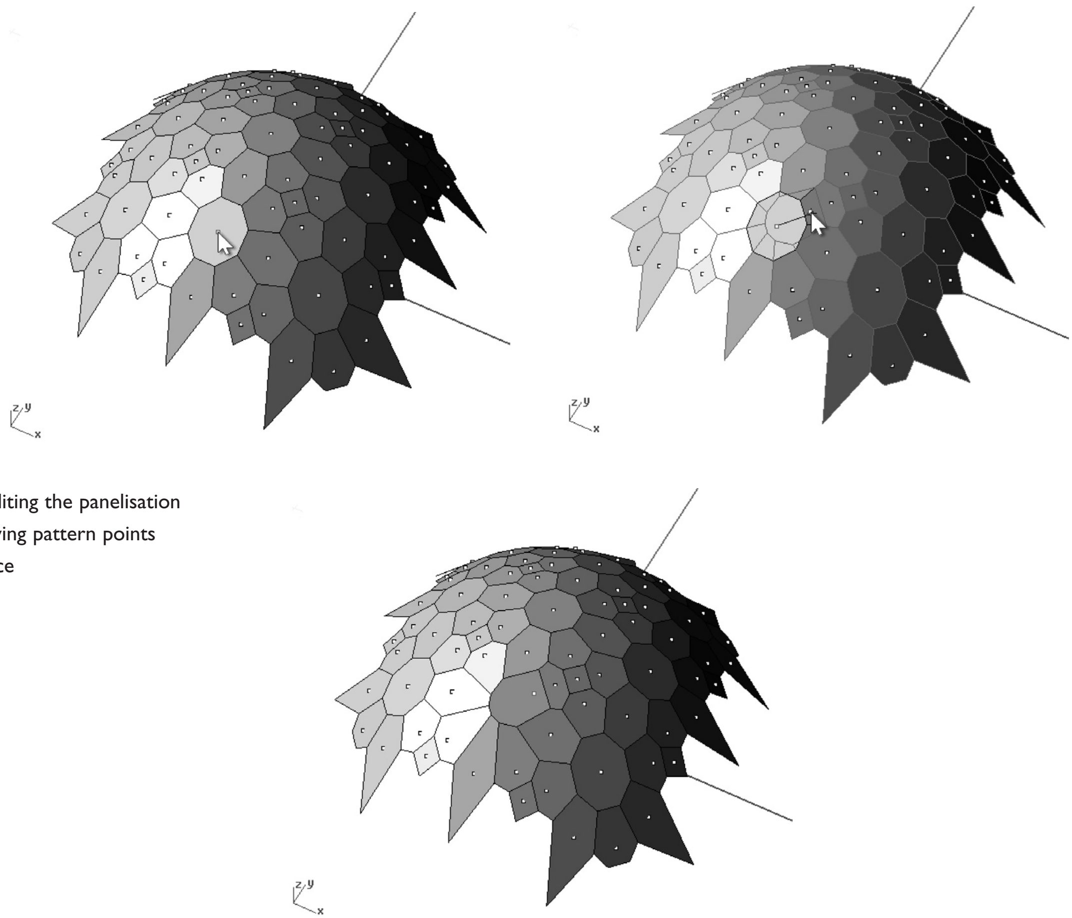


(b) Continuous surface



(c) Generated discrete surface

Once created, the discrete surface can be refined by simply moving the pattern points along the surface (figure 10) and by adding new points or removing existing ones. Manipulating pattern points works just like transforming any other points in Rhino, with the exception that they are constrained to move along the surface. While editing the discrete surface, designers can take advantage of the full range of geometrical modelling aids Rhino offers (simultaneous transformation of multiple objects, object snaps, construction grid, etc.).

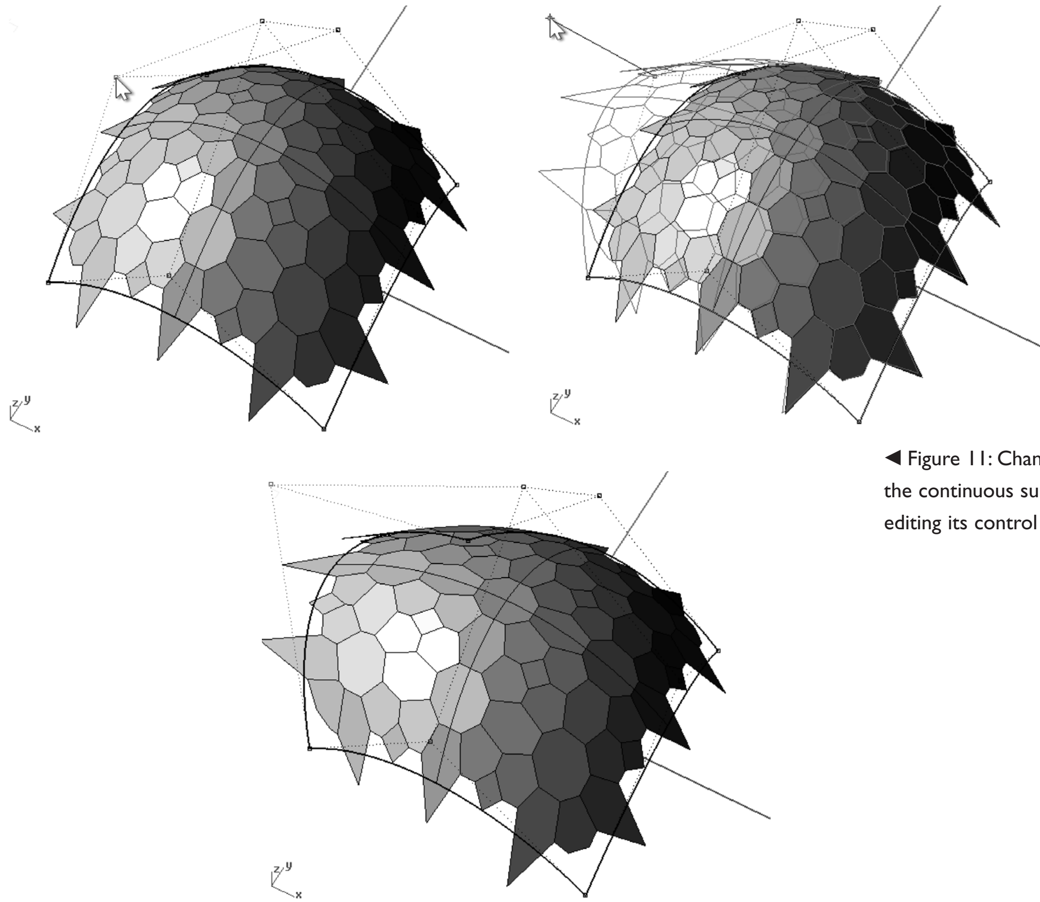


► Figure 10: Editing the panelisation scheme by moving pattern points along the surface

As illustrated in figure 11, the shape of the original continuous surface can still be transformed by editing the control points of the surface subobject.

Because we did not need to introduce any new modelling concepts at all, a designer familiar with Rhino can start working with the tools we created after an apprenticeship of 5 minutes.

Due to the short computation times of the discretisation algorithm, changes to the surface shape and pattern scheme are reflected in real time, which gives designers immediate feedback on their actions – even when working on a shape consisting of thousands of panels. By providing an active geometrical representation of a discrete surface which allows the interactive refinement of designs, we aim to facilitate an explorative design approach and the continual "talk-back" [15] with the design problem. However, relying solely on geometrical constraints is not sufficient in many cases. Therefore we extend our tool kit to Rhino's parametric design environment, Grasshopper. Offering a parametric interface to our algorithm,



◀ Figure 11: Changing the shape of the continuous surface subobject by editing its control points

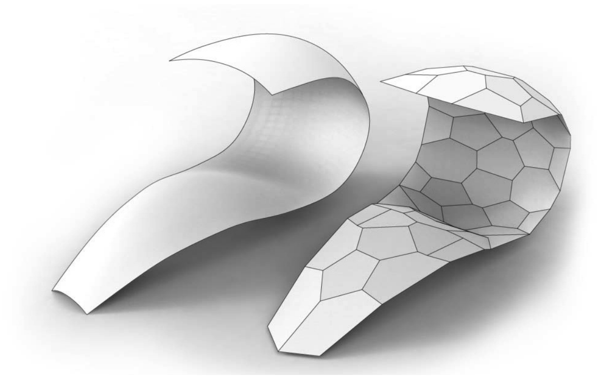
we enable designers to include their own project specific constraints and to potentially enhance their design approach.

4.2. Parametric interface

Although the direct manipulation of geometries using explicit modelling tools is generally perceived to be the most intuitive way of designing with digital tools, algorithmic form-finding techniques are employed by a growing number of digitally advanced practitioners. These techniques require that a design is described by a parametric model instead of a concrete geometric representation. The parametric model thereby describes the process how to arrive at a certain form rather than the form itself.

Additionally to the direct modelling interface we created a Grasshopper component that provides a parametric interface to our algorithm. The component can take either geometrically defined or parametrically generated surfaces and pattern point sets as inputs, it outputs the generated discrete mesh geometry which can then be further processed inside

► Figure 12: Original continuous surface (left) and discrete shape (right) of the prototype

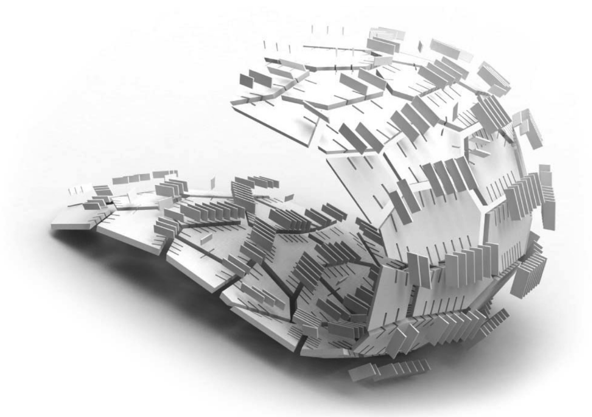


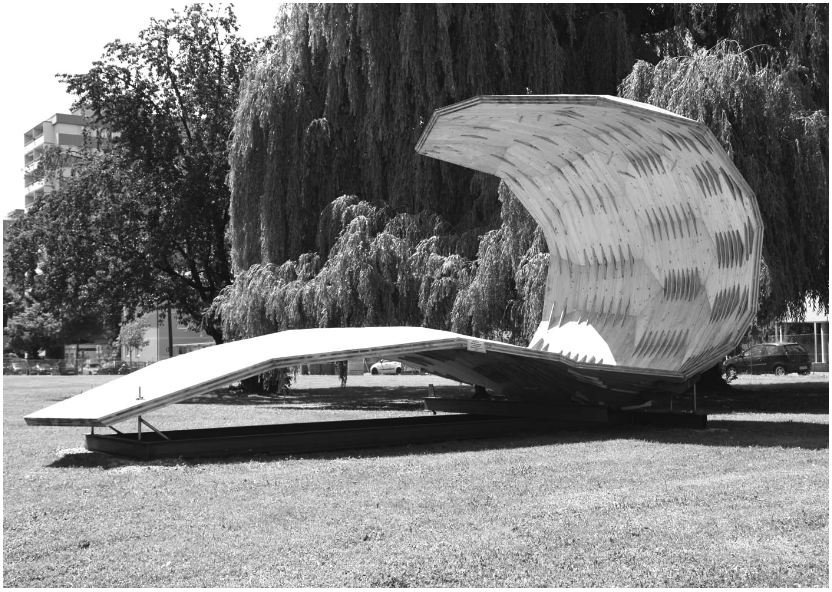
Grasshopper. With this parametric interface we enable designers to build upon our discretisation algorithm and incorporate their own project specific constraints into a parametric model.

We used the parametric interface to create the geometry for a prototype structure that was built earlier this year. The self-supporting shell structure was fabricated from cross-laminated timber (CLT) panels which are connected by wegde-shaped cleats made of Kerto-S – an engineered timber material with high mechanical load capacity. This custom joint system was also developed in the course of our research project and applied for the first time in our prototype structure.

Figure 12 shows the original continuous surface, designed by Emmanuel Ruffo Calderon Dominguez, and the discrete mesh created with our Grasshopper tool. The main challenge here was due to the anisotropic material behavior of timber. Ideally the Kerto cleats should be perfectly in line with the main grain direction, especially in areas of high stresses, since any deviation from the grain direction would weaken the joint. As explicated in much greater detail by Schimek et al.16, the task was to find a panelisation scheme that would allow an optimal load transfer. So the design of the panalisation scheme was mostly performance driven in this case.

► Figure 13: Parametrically generated geometry





◀ Figure 14: The finished prototype structure at the TU campus

After we developed a high-perfoming panalisation scheme we created the three-dimensional panel geometries from the flat polygons of the discrete shape using an offset mesh. The next task was to generate the slots for the Kerto cleats. The number and positions of the joints were determined through a FEM analysis. We implemented a custom Grasshopper component that imported the results of the analysis and directly generated the slot geometries which were then simply cut out of the solid panel BREPs using boolean operations.

Last but not least we created a component that turned the finished panel geometries into tool paths for the milling machine, which completed our fully parametric modelling chain from the initial continuous surface all the way through to generating the data for the fabrication process.

5. CONCLUSION

Tangent plane intersections produce rationalised shapes with a series of desirable characteristics. First of all, discrete meshes generated with the presented method consist solely of polygons that are absolutely planar, which allows non-standard designs to be built from a large variety of industrially mass-produced, standard building materials. Furthermore these planar elements can usually be fabricated efficiently using CNC machinery.

Unlike other discretisation techniques, shapes created with the presented approach can be composed of polygonal facets with varying numbers of sides. The compositions thereby naturally tend to a configuration where only three facets meet at each vertex, which, compared to a triangle mesh, leads to far less complicated construction details.

Beyond the mere practical aspects, and perhaps most notably, tangent plane intersections allow designers to compose complex shapes with a unique ornamental aesthetic. After all, design rationalisation is not just a vehicle for cost reduction but a core architectural matter that can have significant impact on the quality of a design. We are convinced that the most successful results can be achieved if form and rationalisation are developed alongside and in accordance with each other, rather than a posteriori imposing a rationalisation strategy on an already finished design, and therefore conceived a set of tools that can be integrated into the design process from earliest design stages on.

The geometrical method proved to be an excellent basis for the development of efficient and intuitive design tools. Both aesthetic and performative aspects can be controlled by manipulating the simplest parameters – points on a surface, and the ability to perform local changes and the real-time feedback enable an explorative design approach.

Tangent plane intersections are a rationalisation strategy that was well worth investigating. We strongly believe in the potential of this geometrical approach and look forward to continue our research in an upcoming project phase.

ACKNOWLEDGEMENTS

The research was funded by the Austrian Science Fund (FWF) under the grant L695.

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